International Journal of Heat and Mass Transfer 52 (2009) 4244–4253

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Mixed convection with viscous dissipation and pressure work in a lid-driven square enclosure

A. Barletta [a,*], D.A. Nield [b]

[a] Dipartimento di Ingegneria Energetica, Nucleare e del Controllo Ambientale (DIENCA), Università di Bologna, Via dei Colli 16, I–40136 Bologna, Italy
[b] Department of Engineering Science, University of Auckland, Private Bag 92019, Auckland 1142, New Zealand

ARTICLE INFO

Article history:
Received 23 February 2009
Accepted 6 April 2009
Available online 13 May 2009

Keywords:
Mixed convection
Laminar flow
Viscous dissipation
Pressure work
Numerical solution

ABSTRACT

Buoyant laminar flow in a square lid-driven enclosure is analysed. The vertical sides are kept isothermal at different temperatures, while the horizontal sides are insulated. Assisting mixed convection flow due to uniform motion of the top side is considered. The governing balance equations are solved numerically by employing a Galerkin finite element method. The effects of viscous dissipation and pressure work are taken into account. In order to investigate the influence of these effects, the Nusselt number is evaluated with respect to the heat fluxes at both vertical sides, for different values of the Rayleigh number and of the Péclet number based on the lid velocity. Two sample fluids are considered: a gas and a highly viscous liquid. In the framework of the Oberbeck–Boussinesq approximation, a comparison is made between three different energy balance models: (A) enthalpy formulation (pressure work and viscous dissipation are included); (B) internal-energy formulation (viscous dissipation is included); (C) both pressure work and viscous dissipation are neglected. It is shown that, in the absence of a lid motion, the three models yield substantially the same predictions. On the other hand, when the forced flow induced by the lid motion becomes sufficiently large, the three models yield discrepant results, thus implying that pressure work and viscous dissipation are not negligible. Moreover, it is shown that, in this case, model (A) yields unphysical results, while model (B) leads to reasonable predictions.

1. Introduction

The problem of convection in a two-dimensional laterally heated square cavity (with thermally insulated top and bottom) is of great interest to the computational fluid dynamics community (e.g. see de Vahl Davis [1]) as well as being applicable to a wide variety of practical problems. A paper by Mallinson et al. [2] and an associated conference poster presented by Mallinson has stimulated a flurry of recent work on this problem [3–6]. Mallinson raised the question of the effect of viscous dissipation, which is equivalent to a volumetric heat source, on the energy balance and the entropy budget. Costa [3,4] went a step further by claiming that if the viscous dissipation term is taken into account in the energy balance, also the pressure work term needs to be taken into account. This approach has strong implications and, in particular, recognizes a central role of the pressure work contribution in the energy balance of flows with viscous dissipation. This point has been discussed in Nield [5] and in Barletta [6]. The main objection to the arguments presented in [3,4] is based on known results in the case of forced convection, which is a limiting case of mixed convection. With forced convection it is well known [7] that it is possible to have a situation where there is substantial viscous dissipation but negligible pressure work. These known results imply that the viscous dissipation effects may be important, despite a negligible contribution of the pressure work, also in the case of buoyant flows. Otherwise, one would have a substantial jump in the difference between global viscous dissipation and global pressure work as soon as one added some buoyancy, no matter how small the amount.

It is hypothesized by the present authors that the balance between the work done by pressure forces and the energy dissipated by viscous effects does not hold in general. As a step towards testing that hypothesis, we have examined a laterally heated box closed with respect to mass flow and with a moving lid, so that the convection is driven by a mixture of natural convection induced by buoyancy and forced convection induced by the moving boundary.

2. Mathematical model

Let us consider a 2D square enclosure with adiabatic horizontal sides and isothermal vertical sides at different temperatures $T_0 - \Delta T/2$ and $T_0 + \Delta T/2$, with $\Delta T > 0$ (see Fig. 1). The top boundary is in steady horizontal motion with velocity u_0. In the stationary

* Corresponding author.
E-mail addresses: antonio.barletta@mail.ing.unibo.it (A. Barletta), d.nield@auckland.ac.nz (D.A. Nield).

0017-9310/$ - see front matter
doi:10.1016/j.ijheatmasstransfer.2009.04.003

Nomenclature

c_p	specific heat at constant pressure
c_v	specific heat at constant volume
Ec	Eckert number, $\alpha^2/(c_p \Delta T L^2)$
Ee	expansion energy number, Eq. (21)
g	modulus of acceleration due to gravity
Ge	Gebhart number, Eq. (21)
k	thermal conductivity
L	side length of the square cavity
Nu_-, Nu_+	average Nusselt numbers, Eq. (22)
p	pressure
P	dimensionless pressure, Eq. (10)
Pe	Péclet number, Eq. (21)
Pr	Prandtl number, Eq. (21)
Ra	Rayleigh number, Eq. (11)
Rb	Rayleigh–Boussinesq number, Eq. (21)
u_0	lid velocity
u, v	velocity x-component and y-component
U, V	dimensionless velocity X-component and Y-component, Eq. (10)
$U_{\max}$	maximum value of U in the line $X = 0$
T	temperature
T_0	average temperature
T_c, T_h	boundary temperatures
x, y	Cartesian coordinates
X, Y	dimensionless Cartesian coordinates, Eq. (10)

Greek symbols

α	thermal diffusivity, $k/(\rho c_p)$ or $k/(\rho c_v)$
β	volumetric coefficient of thermal expansion
ΔT	difference between the temperatures of the vertical sides
θ	dimensionless temperature, Eq. (10)
μ	dynamic viscosity
ν	kinematic viscosity
ρ	mass density
Ω_{PW}	pressure work parameter, Eq. (23)
Ω_{VD}	viscous dissipation parameter, Eq. (24)

natural convection regime, according to the Oberbeck–Boussinesq approximation, the local mass, momentum and energy balance equations are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \frac{\partial p}{\partial x}, \tag{2}$$

$$\rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) - \frac{\partial p}{\partial y} - \rho[1 - \beta(T - T_0)]g, \tag{3}$$

$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \beta T\left(u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y}\right) + \mu\left\{2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right] + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2\right\}, \tag{4}$$

where the fluid properties ρ, μ, β, c_p, k are considered as constants evaluated at the reference temperature T_0.

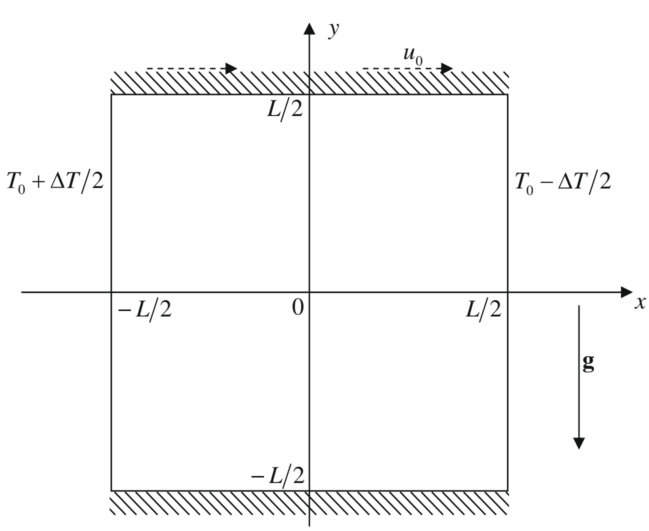

Fig. 1. Drawing of the square cavity.

Boundary conditions are:

$$u = v = 0 \quad \text{at} \quad x = \pm\frac{L}{2} \quad \text{and} \quad y = -\frac{L}{2}, \tag{5}$$

$$u = u_0 \quad \text{and} \quad v = 0 \quad \text{at} \quad y = \frac{L}{2}, \tag{6}$$

$$T = T_0 + \frac{\Delta T}{2} \quad \text{at} \quad x = -\frac{L}{2}, \tag{7}$$

$$T = T_0 - \frac{\Delta T}{2} \quad \text{at} \quad x = \frac{L}{2}, \tag{8}$$

$$\frac{\partial T}{\partial y} = 0 \quad \text{at} \quad y = \pm\frac{L}{2}. \tag{9}$$

2.1. Dimensionless equations

Let us define the dimensionless quantities

$$U = \frac{uL}{\alpha}, \quad V = \frac{vL}{\alpha}, \quad X = \frac{x}{L}, \quad Y = \frac{y}{L},$$
$$P = \frac{(p + \rho g y)L^2}{\rho\alpha^2}, \quad \theta = Ra\,\frac{T - T_0}{\Delta T}, \tag{10}$$

where

$$Ra = \frac{g\beta\Delta T L^3}{\alpha\nu}. \tag{11}$$

Then, Eqs. (1)–(9) can be rewritten as

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \tag{12}$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = Pr\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) - \frac{\partial P}{\partial X}, \tag{13}$$

$$U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = Pr\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} + \theta\right) - \frac{\partial P}{\partial Y}, \tag{14}$$

$$U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} + (Rb + \theta)\left[Ee\left(U\frac{\partial P}{\partial X} + V\frac{\partial P}{\partial Y}\right) - Ge\,V\right] + Ge\left\{2\left[\left(\frac{\partial U}{\partial X}\right)^2 + \left(\frac{\partial V}{\partial Y}\right)^2\right] + \left(\frac{\partial U}{\partial Y} + \frac{\partial V}{\partial X}\right)^2\right\}, \tag{15}$$

$$U = V = 0 \quad \text{at} \quad X = \pm\frac{1}{2} \quad \text{and} \quad Y = -\frac{1}{2}, \tag{16}$$

$$U = Pe \quad \text{and} \quad V = 0 \quad \text{at} \quad Y = \frac{1}{2}, \tag{17}$$

$$\theta = \frac{Ra}{2} \quad \text{at} \quad X = -\frac{1}{2}, \tag{18}$$

$$\theta = -\frac{Ra}{2} \quad \text{at} \quad X = \frac{1}{2}, \tag{19}$$

$$\frac{\partial\theta}{\partial Y} = 0 \quad \text{at} \quad Y = \pm\frac{1}{2}. \tag{20}$$

In Eqs. (12)–(20), the dimensionless parameters

$$Pr = \frac{\nu}{\alpha}, \quad Pe = \frac{u_0 L}{\alpha}, \quad Ee = \frac{\beta\alpha^2}{c_p L^2}, \quad Rb = Ra\,\frac{T_0}{\Delta T}, \quad Ge = \frac{\beta g L}{c_p}, \tag{21}$$

have been used. Here Pr, Pe and Ge are the familiar Prandtl, Péclet and Gebhart numbers. We propose that Rb be called the Rayleigh–Boussinesq number. We also suggest that Ee be called the expansion energy number. We have in mind that α/L is a diffusion velocity, so that $(\alpha/L)^2$ may be regarded as a diffusion energy per unit mass, while c_p/β may be regarded as an expansion energy per unit mass.

The importance in the present context of the choice of the reference scales used in scaling the physical variables has been emphasized by Nield [5]. In particular, Nield noted that Costa in Refs. [3] and [4] had scaled his velocity in terms of α/L, the conduction scale. Nield argued that the appropriate velocity scale for strong convection in a porous medium was $(\alpha/L)Ra^{1/2}$ while that for weak convection was $(\alpha/L)Ra$. Since by weak convection we mean $Ra = O(1)$, the scale for weak convection is effectively the same as that for conduction. In the present problem, the velocity of the lid, namely u_0, is available as an alternative velocity scale. We have chosen α/L as our velocity scale for ease of comparison with the results obtained by other authors. The ratio of the two scales now enters our mixed convection problem as the Péclet number, $Pe = u_0L/\alpha$. A small value of Pe/Ra corresponds to the natural convection limit.

The average Nusselt number can be evaluated both at the left vertical side $X = -1/2$ and at the right vertical side $X = 1/2$, namely

$$Nu_- = -\frac{1}{Ra}\int_{-1/2}^{1/2} \left.\frac{\partial\theta}{\partial X}\right|_{X=-1/2} dY, \qquad Nu_+ = -\frac{1}{Ra}\int_{-1/2}^{1/2} \left.\frac{\partial\theta}{\partial X}\right|_{X=1/2} dY. \tag{22}$$

The overall contribution of the effects of pressure work and viscous dissipation can be evaluated by defining the dimensionless quantities,

$$\Omega_{PW} = \int_{-1/2}^{1/2}\int_{-1/2}^{1/2} (Rb + \theta)\left[Ee\left(U\frac{\partial P}{\partial X} + V\frac{\partial P}{\partial Y}\right) - Ge\,V\right] dX\,dY, \tag{23}$$

$$\Omega_{VD} = Ge\int_{-1/2}^{1/2}\int_{-1/2}^{1/2} \left\{2\left[\left(\frac{\partial U}{\partial X}\right)^2 + \left(\frac{\partial V}{\partial Y}\right)^2\right] + \left(\frac{\partial U}{\partial Y} + \frac{\partial V}{\partial X}\right)^2\right\} dX\,dY. \tag{24}$$

A relationship between Nu_-, Nu_+, Ω_{PW} and Ω_{VD} can be obtained by integrating both sides of Eq. (15) over the whole enclosure, taking into account Eq. (12), applying Gauss theorem, utilizing the condition of zero normal component of velocity on the four boundary sides. Thus, from Eq. (22), one obtains

$$(Nu_+ - Nu_-)Ra = \Omega_{PW} + \Omega_{VD}. \tag{25}$$

In the case $Pe = 0$, no work input is supplied to the fluid. Then, the first law of thermodynamics prescribes that, in the stationary regime, no net heat flux can be exchanged through the enclosure boundary. This conclusion allows one to infer that the right hand side of Eq. (25) should be zero. As a consequence, Nu_- and Nu_+ should be coincident and $\Omega_{PW} = -\Omega_{VD}$. However, the Oberbeck–Boussinesq approximation implies imperfect local balances and, hence, may yield a lack of compensation between the heat fluxes on the two isothermal sides [6]. In this respect, it is worth mentioning that Eq. (25) is expected to be satisfied by the numerical solution within its accuracy. On the other hand, the requirement of the first law, $Nu_- = Nu_+$, may not be satisfied since the local energy balance is not an exact one.

In the case $Pe \neq 0$, work is supplied to the fluid through the moving top boundary. By the first law of thermodynamics, this work implies different heat fluxes through the vertical isothermal boundaries and, hence, a nonvanishing difference $Nu_+ - Nu_-$.

2.2. Chandrasekhar's approach

A widely employed simplification of the energy balance consists in neglecting both the viscous dissipation effect and the pressure work effect. This approach is recovered when one sets $Ee = 0$ and $Ge = 0$ in Eq. (15).

A different statement of the Oberbeck–Boussinesq approximation is that proposed by Chandrasekhar [8] and recently discussed by Barletta [6]. Chandrasekhar's approach to the Oberbeck–Boussinesq approximation is based on the internal-energy formulation of the local energy balance. As explained in several textbooks on convective heat transfer (see for instance Arpaci and Larsen [9]), there are two possible formulations of the local energy balance in a fluid. The first is the enthalpy formulation, based on the specific heat at constant pressure c_p and, according to the Oberbeck–Boussinesq approximation, is expressed by our Eq. (4). The second is the internal-energy formulation, based on the specific heat at constant volume c_v and, according to the Oberbeck–Boussinesq approximation, is expressed by

$$\rho c_v\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \mu\left\{2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right] + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2\right\}. \tag{26}$$

The dimensionless counterpart of Eq. (26) is

$$U\frac{\partial\theta}{\partial X} + V\frac{\partial\theta}{\partial Y} = \frac{\partial^2\theta}{\partial X^2} + \frac{\partial^2\theta}{\partial Y^2} + Ge\left\{2\left[\left(\frac{\partial U}{\partial X}\right)^2 + \left(\frac{\partial V}{\partial Y}\right)^2\right] + \left(\frac{\partial U}{\partial Y} + \frac{\partial V}{\partial X}\right)^2\right\}, \tag{27}$$

provided that the dimensionless parameters defined in Eq. (21) are rearranged by introducing c_v instead of c_p and redefining the thermal diffusivity α accordingly. Interestingly enough, Eq. (26) is free of any contribution due to the pressure work from the very beginning, by simply claiming that the flow field is solenoidal as implied by the Oberbeck–Boussinesq approximation.

It must be mentioned that the formulation expressed by Eq. (26) is supported by the discussion of the energy balance for buoyant flows carried out by Landau and Lifshitz [10] and by Bejan [11], with an important difference: the specific heat is not c_v but c_p. Although important, this difference will not influence the comparison between the predictions due to Eq. (4) and those due to Eq. (26) carried out in the following sections. In fact, the use of either c_p or c_v becomes hidden in the dimensionless Eq. (27). Moreover, the really crucial feature is that Eq. (27) contains the source term due to the viscous dissipation, but not the source term due to the pressure work.

Obviously, by employing Eqs. (27) and (25) would be replaced by

$$(Nu_+ - Nu_-)Ra = \Omega_{VD}. \tag{28}$$

Eq. (28) implies that, in every case, $Nu_+ > Nu_-$ as Ω_{VD} is a positive quantity unless the fluid is at rest.

2.3. Ranges of the parameters

Among the six governing parameters defined in Eqs. (11) and (21), only Ra and Pe are influenced by the boundary conditions, namely by the values of u_0 and ΔT. The other four parameters, Pr, Ge, Rb, Ee, depend uniquely on the reference temperature T_0, on the width of the enclosure and on the properties of the fluid. In the following, different convection regimes are investigated by considering different values of the pair (Ra,Pe). On the other hand, the values of Pr, Ge, Rb, Ee will be fixed according to two possible sample cases:

$$\text{Gas}: \ Pr = 0.71, \ Ge = 10^{-5}, \ Pr\,Rb = 10^{7}, \ Ee = 10^{-12}; \tag{29}$$

$$\text{Highly viscous liquid}: \ Pr = 10^{3}, \ Ge = 10^{-6}, \ Pr\,Rb = 10^{11}, \ Ee = 10^{-18}. \tag{30}$$

Table 1
Mesh independence test and comparison with benchmark results in the case $Ra = 10^6$, $Pe = 0$, $Ge = 0$ and $Ee = 0$.

Number of elements	Nu_-	Nu_+	U_{max}
13,160	8.8261	8.8260	64.84
17,866	8.8256	8.8256	64.83
23,524	8.8254	8.8254	64.83
33,276	8.8253	8.8253	64.83
41,272	8.8253	8.8253	64.83
Results obtained by other authors			
de Vahl Davis [1]	8.817	–	64.63
Le Quéré [13]	8.825	–	64.83
Syrjälä [14]	8.8251	–	64.8330
Leal et al. [15]	8.826	–	64.83

The reason for prescribing round values of $Pr\,Rb$ instead those of Rb is that, for a gas, one has $\beta T_0 = 1$. Moreover, it is easily verified that $\beta T_0 = Pr\,Rb\,Ee/Ge$.

In order to test the numerical method against existing benchmark solutions obtained in the absence of pressure work and viscous dissipation, a third case will be considered with $Pr = 0.71, Ee = 0$ and $Ge = 0$. In this third case, the value of Rb is unimportant since it does not appear in the governing equations.

The ranges of Ra depend on the choice of a gas or of a highly viscous liquid,

$$\text{Gas}: \quad 0 < Ra \leqslant 10^{6}, \tag{31}$$

$$\text{Highly viscous liquid}: \quad 0 < Ra \leqslant 10^{7}, \tag{32}$$

while, for Pe, the range $0 \leqslant Pe \leqslant 10^3$ is adopted in both cases.

Table 2
Values of Nu_- and Nu_+ for a gas in a thermally driven enclosure ($Pe = 0$).

Ra	Eq. (29)		Eq. (29), no pressure work		$Ge = 0 = Ee$	
	Nu_-	Nu_+	Nu_-	Nu_+	Nu_-	Nu_+
10^2	1.0548	1.0548	1.0015	1.0015	1.0015	1.0015
10^3	1.1604	1.1604	1.1178	1.1178	1.1178	1.1178
10^4	2.2538	2.2538	2.2448	2.2448	2.2448	2.2448
10^5	4.5235	4.5235	4.5216	4.5217	4.5216	4.5216
10^6	8.8258	8.8258	8.8254	8.8254	8.8254	8.8254

Table 3
Values of Nu_- and Nu_+ for a highly viscous liquid in a thermally-driven enclosure ($Pe = 0$).

Ra	Eq. (30)		Eq. (30), no pressure work		$Ge = 0 = Ee$	
	Nu_-	Nu_+	Nu_-	Nu_+	Nu_-	Nu_+
10^2	1.0396	1.0396	1.0015	1.0015	1.0015	1.0015
10^3	1.1482	1.1482	1.1178	1.1178	1.1178	1.1178
10^4	2.2809	2.2809	2.2748	2.2748	2.2748	2.2748
10^5	4.7265	4.7265	4.7255	4.7255	4.7255	4.7255
10^6	9.2310	9.2310	9.2308	9.2308	9.2308	9.2308
10^7	17.347	17.347	17.347	17.346	17.347	17.346

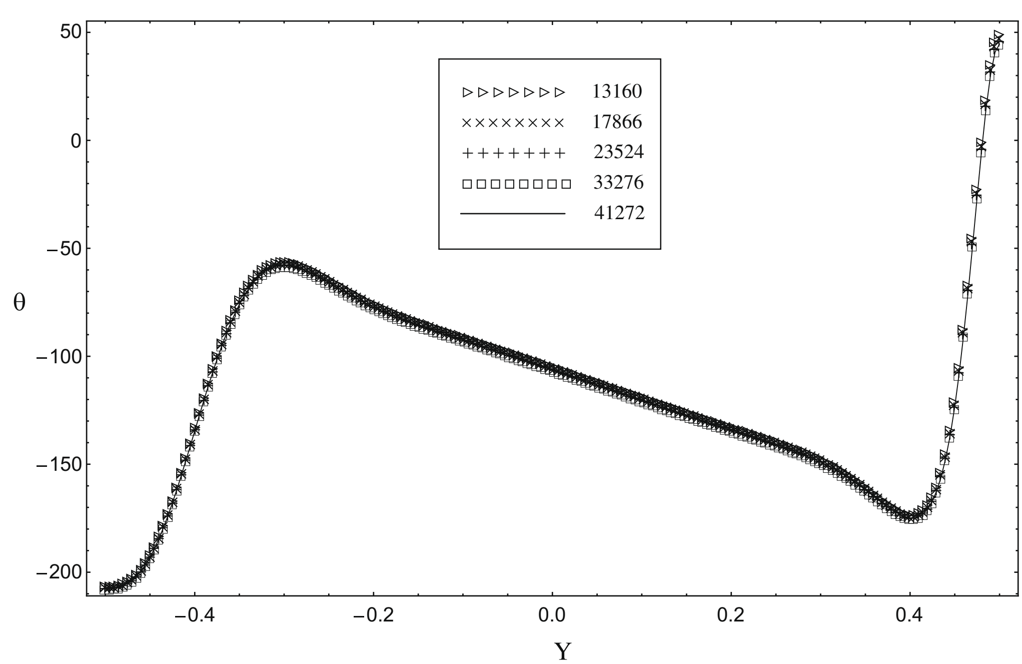

Fig. 2. Mesh independence test: plots of θ versus Y at $X = 0$, for different numbers of mesh elements. Data refer to a gas, Eq. (29), for $Ra = 1000$, $Pe = 1000$ with the local energy balance including both viscous dissipation and pressure work, Eq. (15).

The above determination of reasonable values and ranges for the governing parameters have been made referring to the orders of magnitude of the physical properties of dry air and engine oil and considering $L \sim 10^{-1}$ m.

The special choice $Pe = 0$ corresponds to a purely thermally driven flow.

3. Numerical procedure

The numerical solution of Eqs. (12)–(20) is obtained by a Galerkin finite element method (FEM). Equation-based modelling of convective flows is available through recent commercial FEM packages. Available commercial FEM software packages can solve even rather complex problems quickly and accurately. Moreover, the possibility of performing interactive post-processing and visualization leads to a rather efficient analysis of the results. The FEM software package used in the present paper is Comsol Multiphysics (© Comsol, AB). This software handles local numerical instabilities by adding artificial diffusion through the streamline upwind Petrov–Galerkin method (SUPG) [12].

Unstructured meshes with triangular elements are used. A test of mesh independence is performed with reference to the case of absence of viscous dissipation and pressure work ($Ee = 0$ and $Ge = 0$), with $Pr = 0.71,\ Ra = 10^6$ and $Pe = 0$. Results are reported

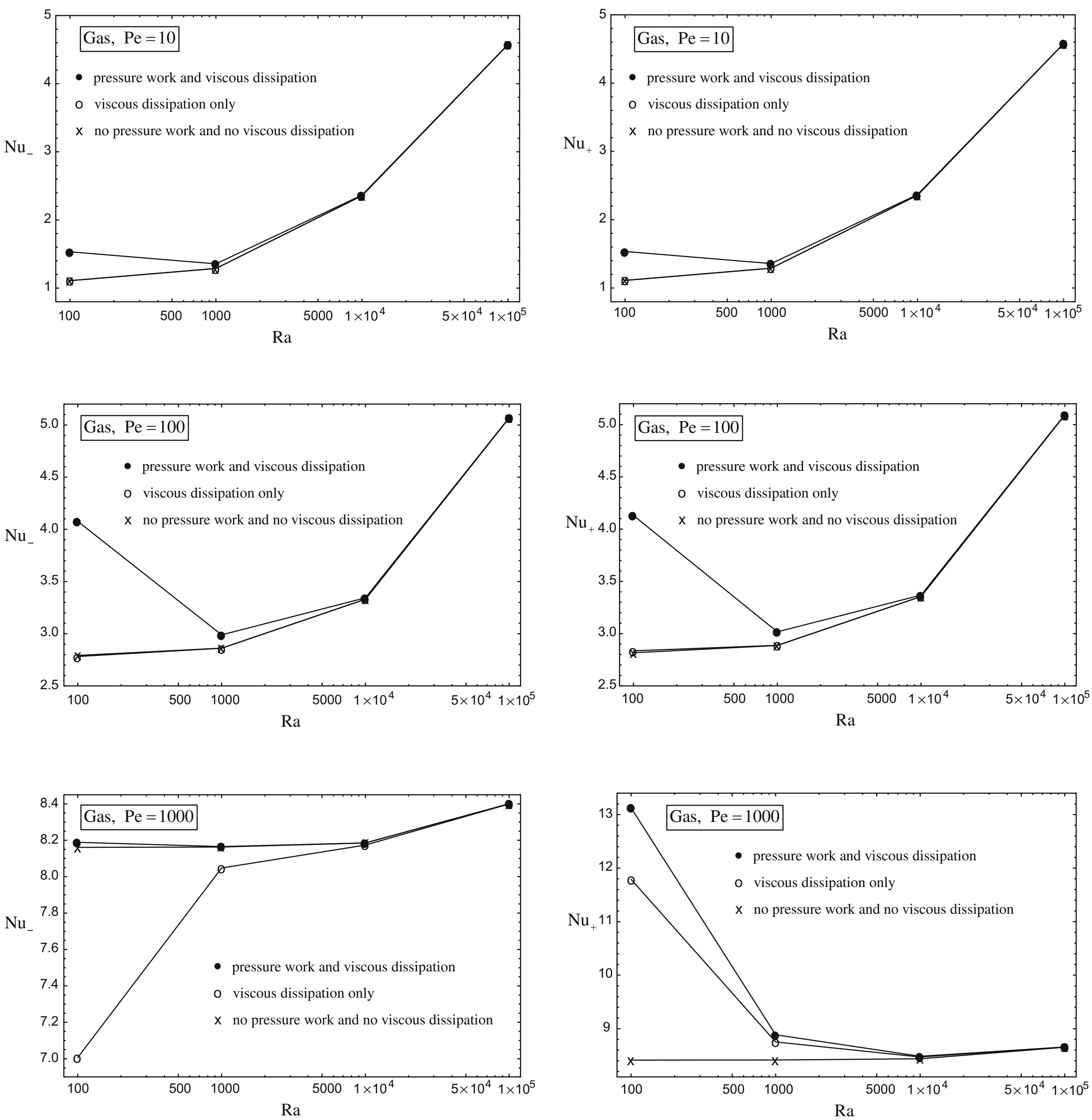

Fig. 3. Plots of $Nu_{\pm}$ versus Ra for a gas, Eq. (29), with increasing values of Pe. The data referring to the model with no pressure work and no viscous dissipation correspond to $Ge = 0 = Ee$. The model including viscous dissipation refers to the energy balance Eq. (27).

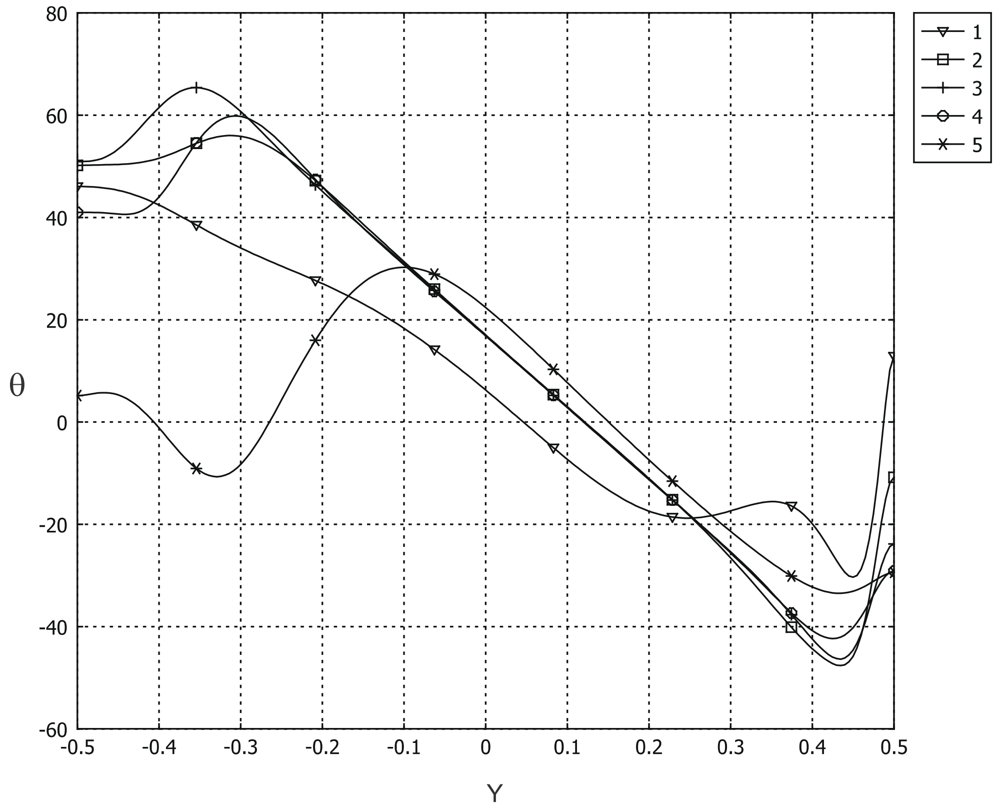

Fig. 4. Plots of θ versus Y, for $X = -0.4$ (line 1), $X = -0.2$ (line 2), $X = 0$ (line 3), $X = 0.2$ (line 4), $X = 0.4$ (line 5). Data refer to a gas, Eq. (29), for $Ra = 100, Pe = 1000$ with the local energy balance including both viscous dissipation and pressure work, Eq. (15).

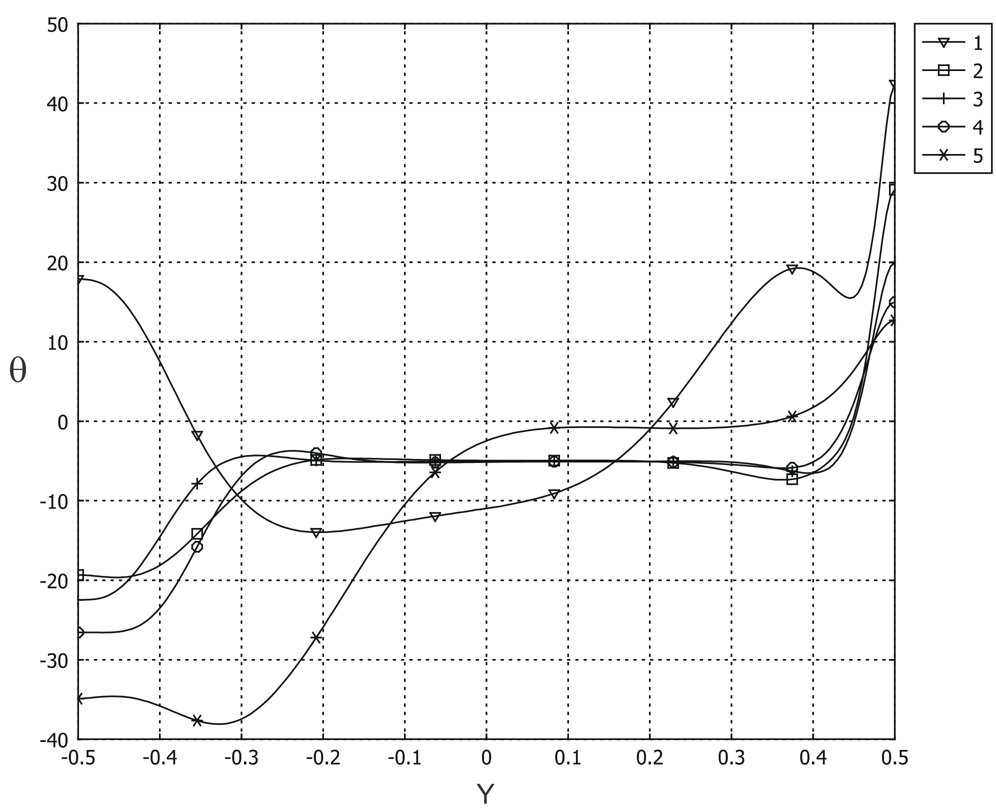

Fig. 5. Plots of θ versus Y, for $X = -0.4$ (line 1), $X = -0.2$ (line 2), $X = 0$ (line 3), $X = 0.2$ (line 4), $X = 0.4$ (line 5). Data refer to a gas, Eq. (29), for $Ra = 100, Pe = 1000$ with the local energy balance including the effect of viscous dissipation, Eq. (27).

in Table 1. Four meshes with increasing refinements are considered sharing the same ratio, 3/20, between the maximum size of the boundary elements and the maximum size of the internal elements. In Table 1, the quantities considered to test mesh independence are Nu_-, Nu_+ and the maximum value of the horizontal velocity component U evaluated on the vertical plane $X = 0$. The latter quantity is denoted by U_{max}. The comparison between the results obtained with the four meshes implies a fair mesh independence of the numerical solution. Moreover, as it is shown in Table 1, the results are in very good agreement with those reported in Refs. [1,13–15].

Fig. 2 displays a mesh independence test referring to a gas, Eq. (29), for $Ra = 1000$ and $Pe = 1000$, with the local energy balance including viscous dissipation and pressure work, Eq. (15). Fig. 2 shows the plots of θ versus Y at $X = 0$, for different numbers of mesh elements. From this figure, one can conclude that the effect of the mesh size on the temperature distribution in the vertical plane $X = 0$ is definitely small.

On account of the above tests, the calculations performed in the following were obtained by means of the mesh with 23,524 elements.

4. Discussion of the results

4.1. Thermally driven cavity ($Pe = 0$)

For the results in Tables 2 and 3 we see that when $Pe = 0$ neither viscous dissipation nor pressure work has a significant effect on the heat transfer, for the realistic data values given in Eq. (29)

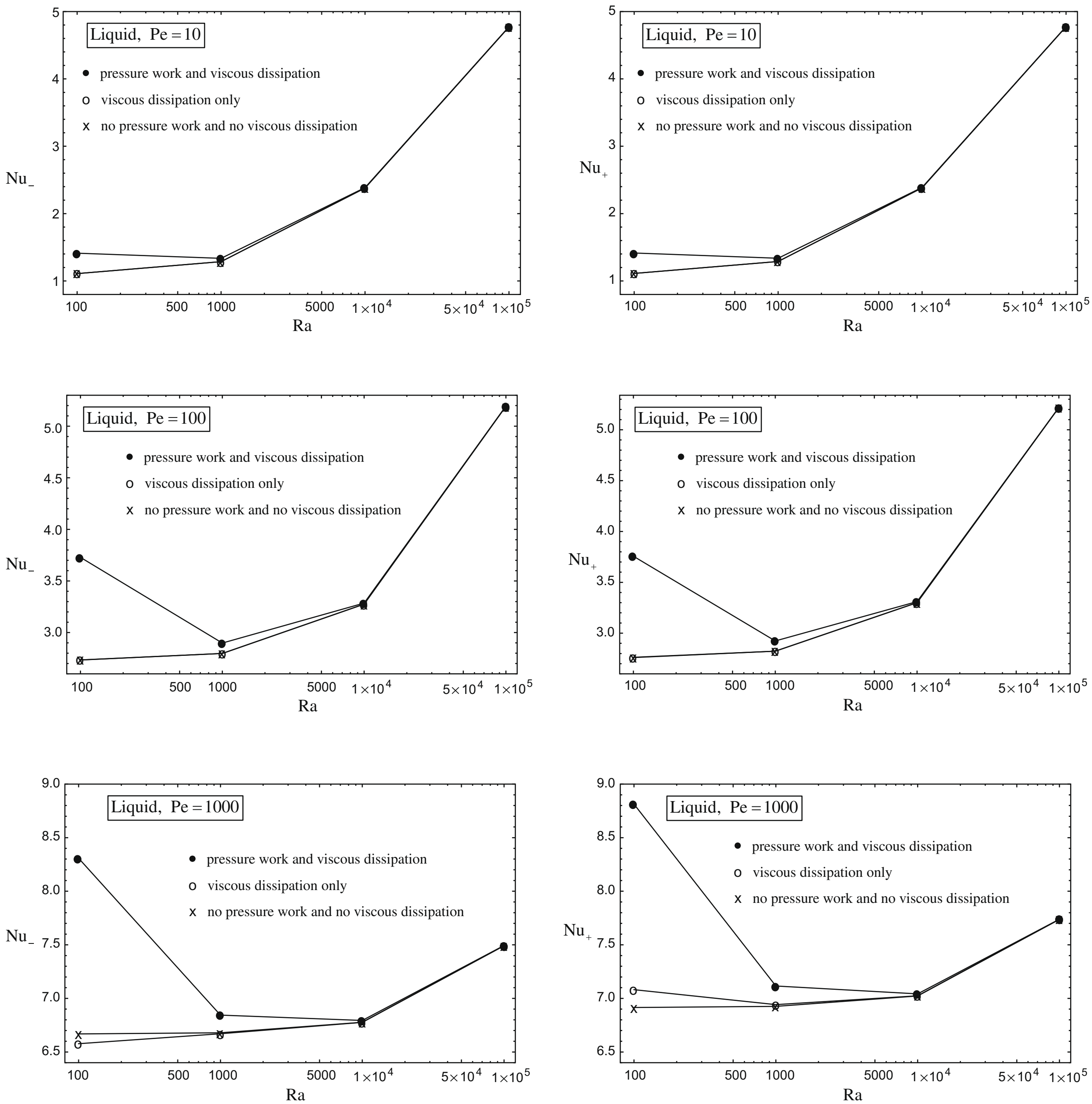

Fig. 6. Plots of $Nu_\pm$ versus Ra for a liquid, Eq. (30), with increasing values of Pe. The data referring to the model with no pressure work and no viscous dissipation correspond to $Ge = 0 = Ee$. The model including viscous dissipation refers to the energy balance Eq. (27).

or (30). In fact, when one substitutes the parameter values from Eq. (29) or (30) in Eq. (15), one immediately sees that the viscous dissipation term is very small. Also, the only pressure work term that is not very small is the term $Rb\,Ge\,V$. This is the term coming from the hydrostatic pressure gradient. From the symmetry of the geometry, one might expect that the average value of V over the fluid domain would be close to zero. Hence the null result might be expected. This follows from the following reasoning. When one drops the small terms, Eq. (15) reduces to

$$U\frac{\partial\theta}{\partial X}+V\frac{\partial\theta}{\partial Y}=\frac{\partial^2\theta}{\partial X^2}+\frac{\partial^2\theta}{\partial Y^2}-Ge\,Rb\,V. \tag{33}$$

Then, the transformation $(X,Y,U,V,\theta)\rightarrow(-X,-Y,-U,-V,-\theta)$ leaves the differential Eqs. (12)–(15) and the boundary conditions (16)–(20), with $Pe=0$, invariant. This means that V is antisymmetric with respect to reflection about a diagonal of the square. It follows that when Eq. (33) is integrated over the square the contribution from the last term is zero, by cancellation. In other words, even when $Rb\,Ge$ is large, the contribution of the pressure work (as well as the contribution of the viscous dissipation) is negligible for the realistic parameter values given by Eq. (29) or (30).

The above arguments do not apply to the case of a lid-driven cavity ($Pe>0$) since, although Ge is very small, very high velocity gradients may arise next to the lid when $Pe>0$. The high-strain local conditions can be accompanied by important viscous dissipation effects.

The reason why Costa [3] obtained a huge effect of viscous dissipation and pressure work for the thermally driven square cavity is that he considered unrealistically high values of his parameter $Ec=\alpha^2/(c_p\,\Delta T\,L^2)$. Reasonably practical values of this parameter are of order 5×10^{-11} for a gas and 3×10^{-17} for an engine oil, but instead he considered $Ec=10^{-7},10^{-6},10^{-5}$.

4.2. Lid-driven cavity ($Pe>0$)

Figs. 3 and 6 illustrate the behaviour of Nu_- and Nu_+ versus Ra for different values of Pe. Fig. 3 refers to a gas, Eq. (29), while Fig. 6 refers to a highly viscous liquid, Eq. (30). Both figures show that the discrepancies between the predictions of the three local energy balance models:

(A) energy balance including both pressure work and viscous dissipation, Eq. (15),
(B) energy balance including viscous dissipation, Eq. (27),
(C) energy balance neglecting both pressure work and viscous dissipation, Eq. (15) with $Ge=0=Ee$,

become increasingly evident as Pe increases and Ra decreases. One may easily see that, for $Ra>10^3$, the three energy balance models yield substantially coincident results. This means that, in the range $0\leqslant Pe\leqslant10^3$, the effects of viscous dissipation and pressure work are not very intense as long as $Ra>10^3$. The latter conclusion holds both for the gas and for the highly viscous liquid. On the other hand, discrepancies between models (A), (B) and (C) are especially evident when $Ra=10^2$ and $Pe=10^3$. The occurrence of discrepancies reveals that pressure work and viscous dissipation cannot be neglected and this certainly rules out the validity of the predictions of model (C). Then, the question is: which model among (A) and (B) gives more reliable predictions?

An answer to this question can be given with certainty only by an experimental validation of the results. However, Figs. 4, 5, 7 and 8 give us an important hint. These figures display dimensionless temperature distributions over vertical sections ($X=$ constant) of the enclosure. They refer to a gas (Figs. 4 and 5) or to a highly viscous liquid (Figs. 7 and 8). Figs. 4 and 7 are obtained by using energy balance model (A), while Figs. 5 and 8

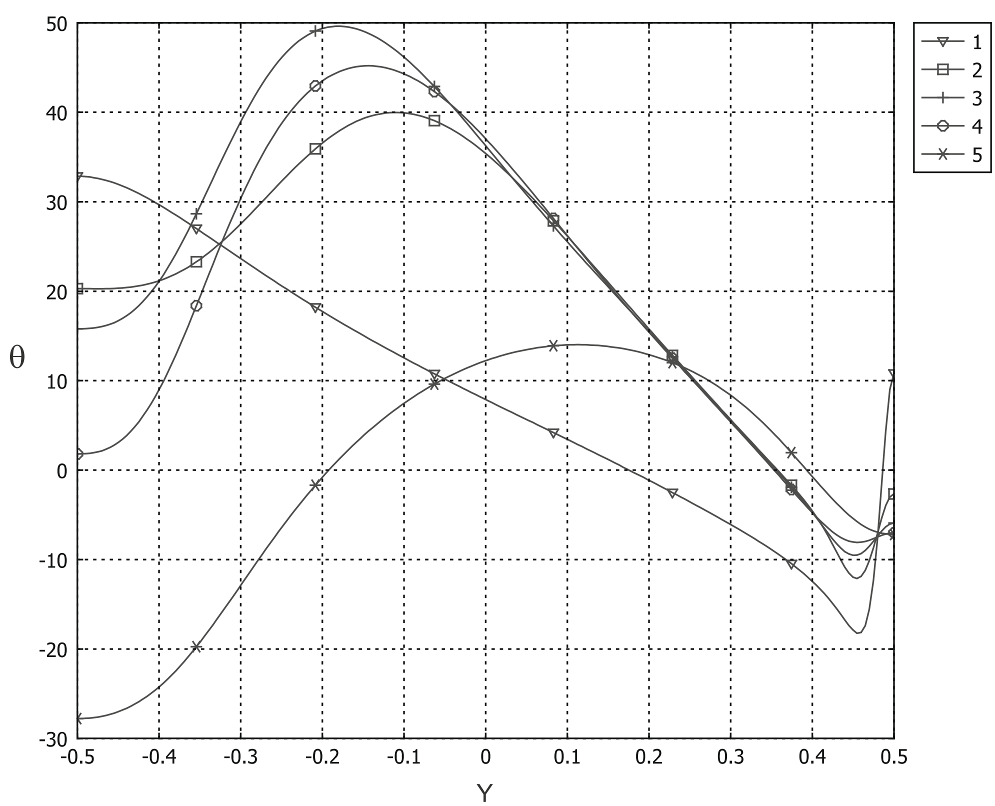

Fig. 7. Plots of θ versus Y, for $X=-0.4$ (line 1), $X=-0.2$ (line 2), $X=0$ (line 3), $X=0.2$ (line 4), $X=0.4$ (line 5). Data refer to a liquid, Eq. (30), for $Ra=100$, $Pe=1000$ with the local energy balance including both viscous dissipation and pressure work, Eq. (15).

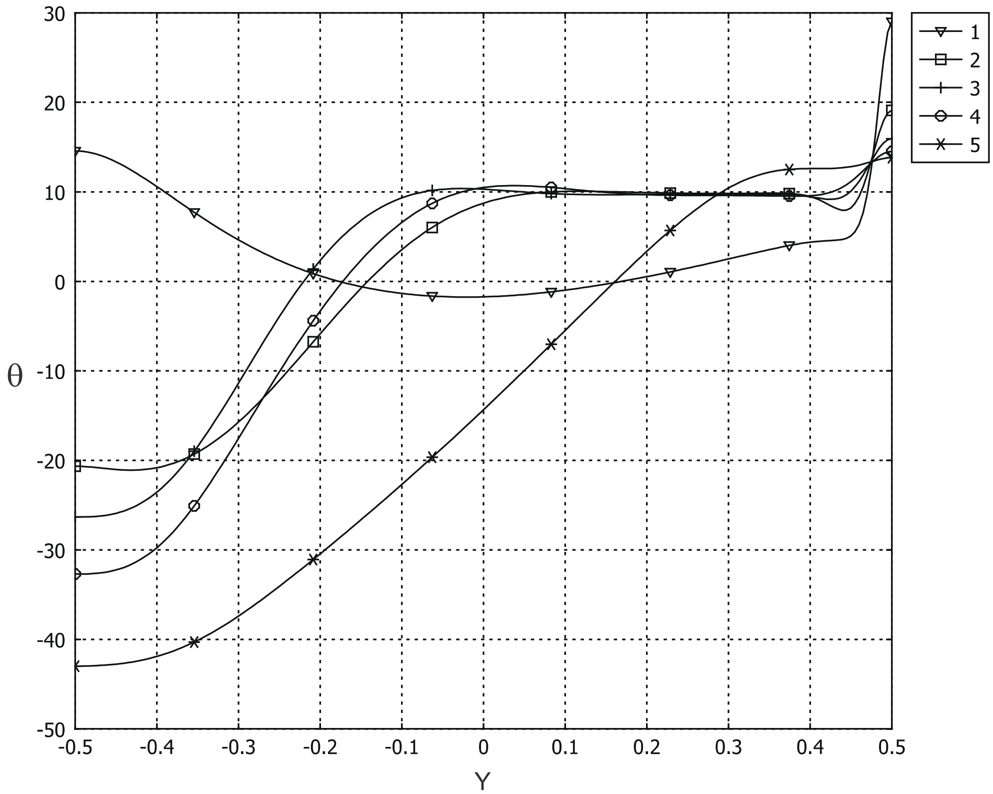

Fig. 8. Plots of θ versus Y, for $X = -0.4$ (line 1), $X = -0.2$ (line 2), $X = 0$ (line 3), $X = 0.2$ (line 4), $X = 0.4$ (line 5). Data refer to a liquid, Eq. (30), for $Ra = 100, Pe = 1000$ with the local energy balance including the effect of viscous dissipation, Eq. (27).

are based on model (B). The most evident information one can infer from Figs. 4 and 7 is that, if one adopts model (A), then one would predict the existence of a bulk of hot fluid placed in the lower half of the enclosure. Apart from the evident problem of thermal instability of this solution, due to the adverse temperature gradient, the prediction appears to be hardly conceivable on a purely physical ground. In fact, the moving lid generates an effect of shear stress and a consequent strain condition in the fluid. This effect is an external forcing that acts on the system and is responsible of the viscous heating effect. Obviously, flow in the enclosure is not only generated by this external forcing, but also by buoyancy. However, buoyancy does not yield in itself an important viscous heating contribution, as it is shown by the results obtained for smaller values of *Pe*. The expected behaviour is to find the hottest part of the fluid next to the moving lid for two reasons: the stress and strain generated by the external forcing are localized next to the lid; the buoyancy effect moves the hot fluid to the top of the enclosure. This expected behaviour is compatible with the predictions of model (B) as can be inferred from Figs. 5 and 8. Thus, one would conclude that model (B) yields results more reliable than those produced by model (A). A similar conclusion has been recently reached by the present authors in the analysis of the fully developed mixed convection flow in a vertical parallel plane channel [16].

5. Conclusions

Combined forced and free flow has been studied in a square enclosure with vertical sides kept at unequal uniform temperatures and with the top lid kept in uniform motion. Assisting conditions are analysed, such that the Péclet number based on the lid velocity is non-negative. The local balance equations, expressed according to the Oberbeck–Boussinesq approximation, have been solved by a Galerkin finite-element method. A comparison has been made between three different energy balance models:

(A) energy balance where both pressure work and viscous dissipation are included,
(B) energy balance where viscous dissipation is included,
(C) energy balance where both pressure work and viscous dissipation are neglected,

in order to show the relative importance of the effects of pressure work and viscous dissipation, as well as to test the reliability of these energy balance models. To compare (A), (B) and (C) through a wide range of different conditions, two rather different sample fluids were considered: a gas and a highly viscous liquid.

The most important results obtained by the above study are the following.

- In the purely buoyant flow case, *i.e.* for a vanishing lid velocity, discrepancies between models (A), (B) and (C) are always decidedly small and practically negligible. The agreement between the different models increases with the Rayleigh number, *Ra*, and becomes almost perfect for $Ra > 10^4$. As a consequence, one is justified in neglecting pressure work and viscous dissipation in this case.
- In the assisting mixed convection flow, *i.e.* for a positive Péclet number, *Pe*, two regimes exist: (*i*) a dominant-buoyancy regime, $Ra > 10^3$, where the influence of both pressure work and viscous dissipation is negligible and models (A), (B) and (C) yield almost the same predictions; (*ii*) a dominant-forcing regime, $Ra < 10^3$ and large values of *Pe* (approximately greater than 10^2), where viscous heating effects due to the lid shear stress become important and models (A), (B) and (C) yield discrepant predictions. The occurrence of important viscous heating effects in the

dominant-forcing regime suggests an inadequacy of model (C). Moreover, model (B) appears to yield more reasonable results than model (A). In fact, the latter would predict lower fluid temperatures in the neighbourhood of the moving upper lid. This prediction appears unphysical as the upper part of the enclosure should be the hottest one in a stationary regime due to the lid-induced friction as well as to buoyancy. Although this expected behaviour turned out to be incompatible with the results obtained from model (A), it has been shown to be compatible with those obtained from model (B).

On the basis of these considerations, the energy model (B) appears to be the most reliable in the analysis of the present flow problem for all the investigated regimes. Model (B) is the Oberbeck–Boussinesq approximation of the local energy balance according to Chandrasekhar's internal-energy formulation.

Challenges for future researches on this subject are: to perform experimental investigations in order to confirm the theoretical conclusions drawn with reference to models (A) and (B); to widen the perspective of the present analysis by testing the behaviour for different conditions of combined forced and free flow. In particular, the second challenge implies the analysis of different temperature and velocity boundary conditions as well as different geometries of the flow system.

References

[1] G. de Vahl Davis, Natural convection of air in a square cavity: a bench mark numerical solution, Int. J. Numer. Meth. Fluids 3 (1983) 249–264.

[2] G. Mallinson, K. Moyle, S. Norris, Volumetric methods for evaluating irreversible energy losses and entropy production with application to bioengineering flows, in: Proc. ICHMT International Symposium on Advances in Computational Heat Transfer, April 19–24, 2004, Norway, CHT-04-262, 2004.

[3] V.A.F. Costa, Thermodynamics of natural convection in enclosures with viscous dissipation, Int. J. Heat Mass Transfer 48 (2005) 2333–2341.

[4] V.A.F. Costa, On natural convection in enclosures filled with fluid-saturated porous media including viscous dissipation, Int. J. Heat Mass Transfer 49 (2006) 2215–2226.

[5] D.A. Nield, The modeling of viscous dissipation in a saturated porous medium, ASME J. Heat Transfer 129 (2007) 1459–1463.

[6] A. Barletta, Comments on a paradox of viscous dissipation and its relation to the Oberbeck–Boussinesq approach, Int. J. Heat Mass Transfer 51 (2008) 6312–6316.

[7] D.A. Nield, A.V. Kuznetsov, M. Xiong, Effects of viscous dissipation and flow work on forced convection in a channel filled by a saturated porous medium, Transp. Porous Media 56 (2004) 351–367.

[8] S. Chandrasekhar, Hydrodynamic and Hydromagnetic Stability, Oxford University Press, 1961.

[9] V.S. Arpaci, P.S. Larsen, Convection Heat Transfer, Prentice-Hall, 1984 (pp. 37 and 38).

[10] L.D. Landau, E.M. Lifshitz, Fluid Mechanics, second ed., Pergamon Press, Oxford, 1987 (Chapter 5).

[11] A. Bejan, Heat Transfer, Wiley, New York, 1993. pp. 227–231.

[12] Comsol Multiphysics Modeling Guide, Version 3.4, Comsol AB, 2007, p. 441.

[13] P. Le Quéré, Accurate solutions to the square thermally driven cavity at high Rayleigh number, Comp. Fluids 20 (1991) 29–41.

[14] S. Syrjälä, Higher order penalty – Galerkin finite element approach to laminar natural convection in a square cavity, Numer. Heat Transfer A 29 (1996) 197–210.

[15] M.A. Leal, J.S. Pérez-Guerrero, R.M. Cotta, Natural convection inside two-dimensional cavities: the integral transform method, Comm. Numer. Meth. Eng. 15 (1999) 113–125.

[16] A. Barletta, D.A. Nield, Combined forced and free convective flow in a vertical porous channel: the effects of viscous dissipation and pressure work, Transp. Porous Media. doi:10.1007/s11242-008-9320-y.